

# Restlessness & Limit

## Mathematics

- ✦ There are no borders for the human mind
- ✦ The limit for mathematicians
- ✦ The abstract nature of the limit concept

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# There are no borders for the human mind

- ✱ There exists a deep contradiction between the perception of real space and time as bounded entities, on the one hand, and our mind's refusal to accept the idea that "nothing else" exists on the other side of any spatial or temporal border, on the other hand. (what was there before the big bang? what is there at the end of our more or less known universe?).
- ✱ The long and tiring transition from a "bounded" number of things to the concept of an infinite set of numbers (Bolzano, Weierstrass) begins with this attempt to understand what we mean with the word "infinity".
- ✱ It was even more difficult to accept the existence of different numerical infinities (numerable, continuous) and to understand what distinguishes one infinity from the other; to the point that few yet understand how the set of rational numbers (fractions) can contain as many elements as the set of positive integers.



# The Limit of Mathematicians

- ✱ The concept of the limit
- ✱ Mathematics and the limit
- ✱ A geometrical example of the limit
- ✱ Geometrical example: further remarks
- ✱ Geometrical example: the method of exhaustion
- ✱ Archimedes and the method of exhaustion

# The concep of the Limit

- ✦ In modern mathematics the **concept of the limit** arises from the twofold requirement to specify the nature of the set of real numbers and to remove the many critiques to the Newtonian definition of the derivative.
- ✦ In **Cauchy's definition** the limit is associated with a function's behaviour when we approach a fixed point or when this point increases indefinitely.
- ✦ A satisfactory mathematical approach to the limit concept and the computational rules appears only at the end of the XIX century.
- ✦ More recently this fundamental concept was introduced in all mathematical fields, not only in the study of functions of several real variables but also in the study of general abstract spaces such as metric and topological spaces.



# The Mathematicians of the Limit

**Gottfried Wilhelm von Leibniz** (1646 – 1716), German philosopher, mathematician, scientist.



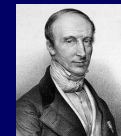
**Sir Isaac Newton** (1643 – 1727), English physicist and mathematician "one of the greatest minds of all time".



**Bernard Placidus Johann Nepomuk Bolzano** (1781 – 1848) Bohemian mathematician, philosopher, logician.



**Augustin-Louis Cauchy** (1789 – 1857), French mathematician and engineer.



**Karl Theodor Wilhelm Weierstrass** (1815 – 1897), German mathematician, "father of modern analysis".



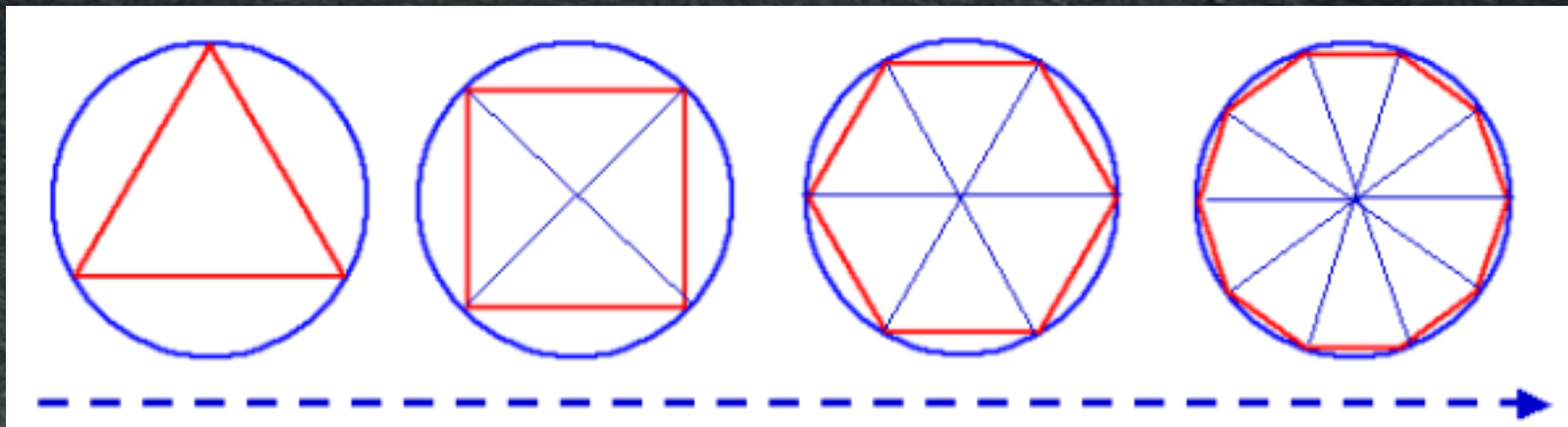
**Ludwig Wittgenstein** (1889 – 1951), Austrian philosopher and logician.



Source: Wikipedia

# A geometrical example of the limit

Consider a polygon inscribed in a circle ...



- When the number of sides increases the polygon looks more and more like the circle.
- If we refer to the polygon as an **n-gon**, where **n** is the number of its sides, we can suggest some mathematical remarks ...



# Geometrical example: further remarks

- ★ As  $n$  increase the  $n$ -gon gets like the circle.
- ★ When  $n$  tends to infinity the  $n$ -gon approaches the circle.
- ★ The  $n$ -gon's limit, when  $n$  tends to infinity, is the circle!

$$\lim_{n \rightarrow \infty} (\mathbf{n\text{-gon}}) = \mathbf{circle}$$

"The  $n$ -gon never identifies with the circle but it gets so near that in practice it can be considered as a circle".

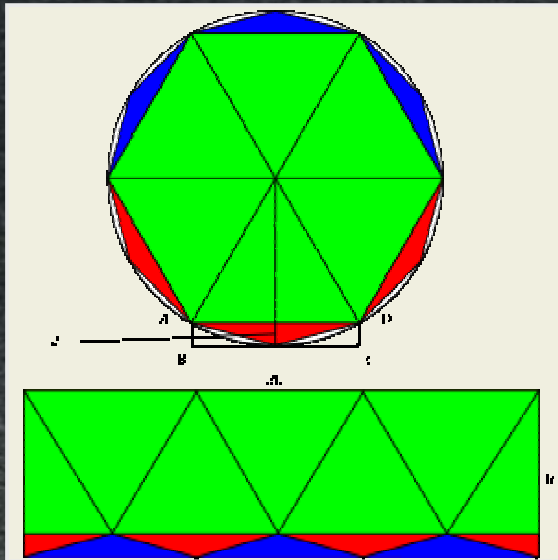
# Geometrical example: the method of exhaustion

- ✱ Consider a circle and all the inscribed  $n$ -gons. As the number of sides increases the  $n$ -gons exhaust the portion of plain occupied by the circle.
- ✱ The area  $A_n$  of each  $n$ -gon is easily computed as the sum of the areas of all the triangles in which it may be divided. When  $n$  increases indefinitely the areas  $A_n$  approach what we shall call the area of the circle.
- ✱ Mathematicians say that, when  $n$  tends to infinity, the areas  $A_n$  tend towards the area  $A$  of the circle and they write

$$\lim_{n \rightarrow \infty} A_n = A$$

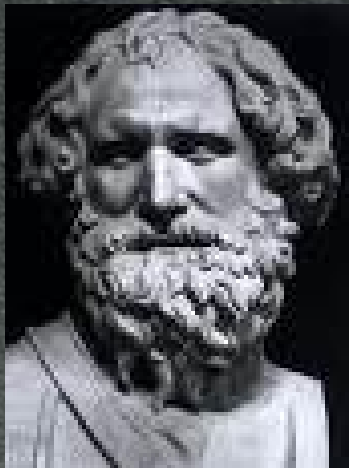


# Archimedes and the method of exhaustion



About 2300 years ago Archimedes (287-212 a.C.) used this idea: by computing the areas of the first  $n$ -gons, he obtained an excellent approximation for the area of the circle. In this way he found the first two decimals of the number  $\pi$

$$\pi = 3,14159265358979 \dots$$





The method of exhaustion that Archimedes described in **The Method** represents the basis for the concept of the integral developed by Newton and Leibniz in the XVII century.

# The abstract nature of the limit concept

- ✦ Abstract spaces
- ✦ Painting the derivative
- ✦ Infinite and infinitesimal

Source: Calculus has practical applications, such as understanding the true meaning of the infinitesimals.  
(Image concept by Dr. Lachowska, MIT)

The importance of understanding the infinitesimals ...	
	
How much is a drop of lemonade? I'll give you a drop for nothin'	Can I have a cup full of drops? .



# Abstract Spaces

- ✱ The abstract nature of Cauchy's definition of the limit gains new value only when it is extended to abstract spaces and anyway it doesn't seem to overcome the doubts regarding the definition of the derivative.
- ✱ Infact Newton's and Leibniz's approach to differential calculus was opposed by other scholars and among them by Karl Marx.

# Definition of the derivative

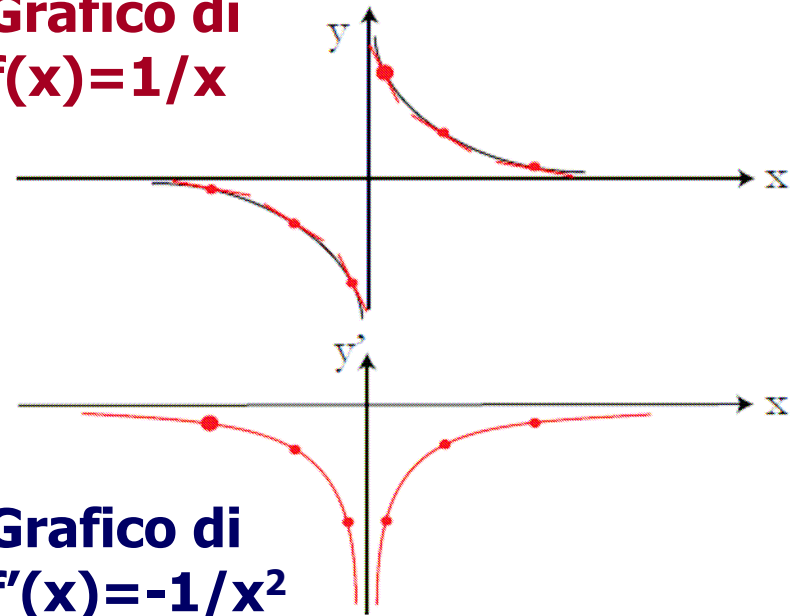
- ✦ Actually the definition of the derivative given by Newton presents an obvious inconsistency. If we consider the ratio (mean velocity) between the increase  $\Delta s$  of the quantity  $s$  (distance covered) and the corresponding increase  $\Delta t$  of the variable  $t$  (time taken), it has sense only if the denominator  $\Delta t$  is different from zero.
- ✦ On the other hand, simple algebraic computations show that the ratio can always be transformed so that we can put  $\Delta t = 0$  and so get the “derivative” (instantaneous velocity) of the quantity  $s$ . In other words, we accept a posteriori an operation which a priori had been ruled out.



# Painting the derivative ...

The first figure gives us the value of the derivative at each point: it is the slope of the tangent line to the function's graph, where the tangent line in a point is defined as "the limit position" of all straight lines passing through that point. Here we have the derivative according to Newton's definition, that Cauchy made rigorous by introducing the limit of the ratio  $\Delta s / \Delta t$ .

**Grafico di**  
 **$f(x)=1/x$**



**Grafico di**  
 **$f'(x)=-1/x^2$**

In the figure below (following Marx's approach) the derivative is an "operator", i.e. a mathematical instrument that associates to any given function another function according to a certain algorithm. In our case, the given function is  $1/x$  and we associate its "derivative function"  $-1/x^2$

# Infinite & Infinitesimal

- ☀ The concepts of “infinitesimal = point” and of “infinite = beyond any bound” suggest a similarity with the identification between the infinitely small and the infinitely big that appears in Hebrew mystic literature.
- ☀ This remark induces to build a bridge between mathematics, logic and philosophy (already existing since a long time, e.g. Wittgenstein’s work).

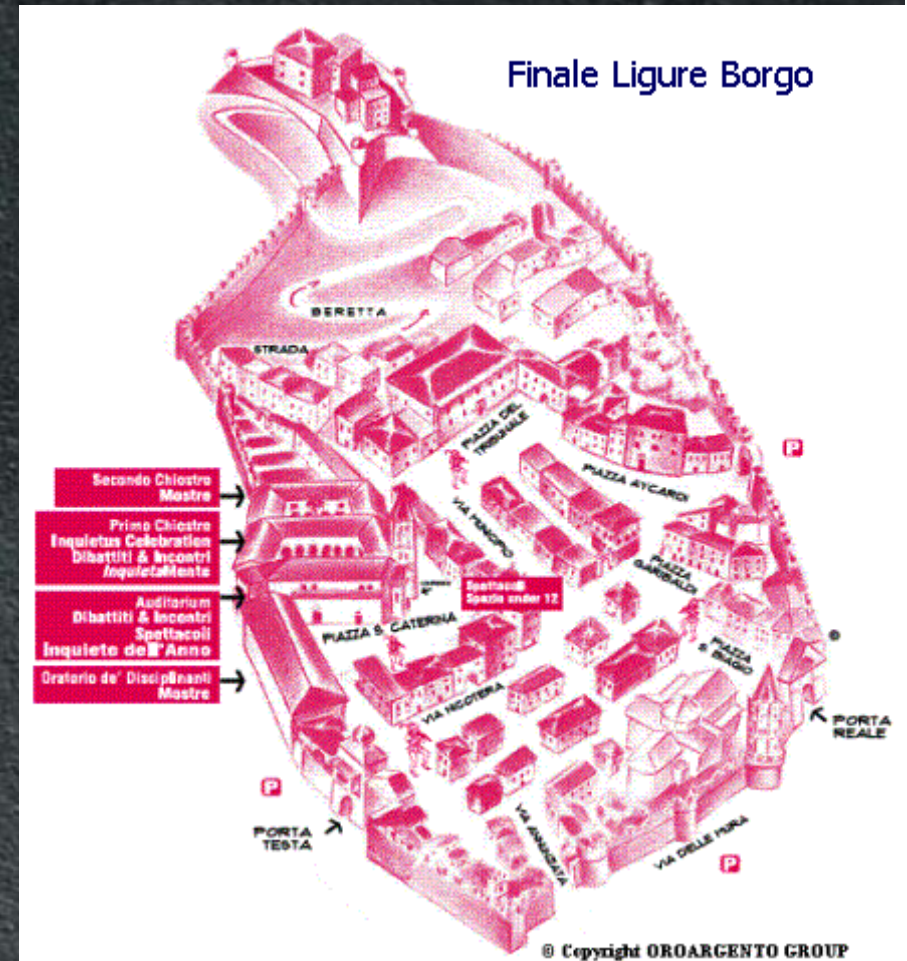




# Places of the Festa dell'Inquietudine 2010

The places of the “Festa” are

- ✦ the Cloisters and Auditorium of Santa Caterina Monastery
- ✦ the Halls of Columns and Arches of the Oratory of Disciplinanti
- ✦ the squares of the Old town of Finale Ligure Borgo (Finalborgo)





# Final Borgo Complex of Santa Caterina



The place name Final Borgo derives from Burgum Finarii, a border town (*ad fines, at the border*) at the time of the Romans and administrative centre of the marquisate of the Del Carretto family between the 14th and 16th centuries. Closed in between medieval walls and still well preserved, interspersed with semi-circular towers and interrupted only by the gates, Borgo di Finale (so called to differentiate it from the Marina) immediately offers the visitor a feeling of protection and welcome.



# Festa dell'Inquietudine 2010 organization



## Promotional Committee:

- ☀ Comune di Finale Ligure
- ☀ Fondazione A. De Mari - Cassa di Risparmio di Savona
- ☀ Provincia di Savona

Planning and organization:  
Circolo degli Inquieti di Savona



# Circolo degli Inquieti



## ☀ Member profile:

- Temperament emotional and imaginative, and at the same time self-critical. Ill suited for conformity to rigid rule.
- Cultural traveller always available to leave for unusual destinations.
- Develop and sustain a lifelong desire for knowledge. Maintain a Socratic ignorance. Know and develop yourself. Be pervaded by doubts.
- Aim at understanding others and their differences.
- Be aware of well-known and knowable matters. Perceive magic and mystery.
- Embark on new adventures and initiatives.

## ☀ Club motto: "The more I understand, the more I do not know", philosopher Tommaso Campanella.



# Events

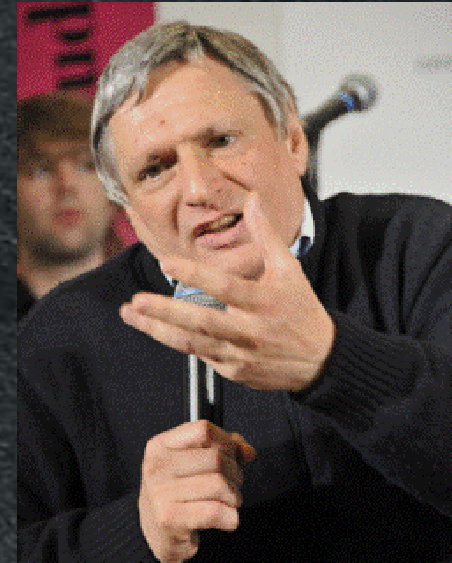
- ★ **Debates & meetings:** Promotion of restlessness as a condition of being human and a synonym of knowledge and cultural growth.
- ★ **Exhibitions & Shows:** Proposition differing aspects of artistic creativity.
- ★ ***InquietaMente*:** Innovative projects dedicated to young people, work and businesses.
- ★ **Inquietus Celebration (IV edition):** "Celebration" of restless personalities who have distinguished themselves for their high intellectual and emotional vitality in specific areas of human activity.
- ★ **Inquietus of the Year (XIII edition):** Celebration of personality that has stood out for being restless.

# Inquietus of the year

<b><i>“The Year”</i></b>	<b><i>Edition</i></b>	<b><i>Celebration</i></b>	<b><i>Inquietus of the year</i></b>
<b>2009</b>	XIII	2010	<b>?</b>
<b>2008</b>	XII	2009	<b>Don Luigi Ciotti</b>
<b>2007</b>	XI	2008	<b>Milly &amp; Massimo Moratti</b>
<b>2006</b>	X	2007	<b>Raffaella Carrà</b>
<b>2005</b>	IX	2006	<b>Règis Debray</b>
<b>2004</b>	VIII	2005	<b>Costa Gavras</b>
<b>2003</b>	VII	2004	<b>Oliviero Toscani</b>
<b>2002</b>	VI	2003	<b>Barbara Spinelli</b>
<b>2001</b>	V	2002	<b>Antonio Ricci</b>
<b>2000</b>	IV	2001	<b>Gino Paoli</b>
<b>1998</b>	III	1999	<b>Francesco Biamonte</b>
<b>1997</b>	II	1998	<b>Gad Lerner</b>
<b>1996</b>	I	1997	<b>Carmen Llera Moravia</b>



# Inquietus of the year 2008 (XII ed)



# Citations & Link

- ✦ The logo of the “Circolo degli Inquieti” was designed by Ugo Nespolo [www.nespolo.com](http://www.nespolo.com)
- ✦ Logo of the “Festa dell’Inquietudine” by Oliviero Toscani & La Sterpaia [www.lasterpaia.it](http://www.lasterpaia.it)
- ✦ Pictures by Emilio Rescigno [www.emiliorescigno.it](http://www.emiliorescigno.it)
- ✦ Presentation background: Ardesia, Pietra di Liguria. “Slate in Liguria: One of the most striking features of Liguria is the extent to which slate is used: the dappled grey roofs, the resorts along the Riviera, the region's medieval churches and their black and white striped facades, the homes of the aristocracy with their grand slate stairways, overdoor decorations, ... wherever you look this fascinating stone has left its mark on the region's history and everyday life”, [www.portale-ardesia.com](http://www.portale-ardesia.com)



## INQUIETI CHANNELS ...



[www.festainquietudine.it](http://www.festainquietudine.it)  
*Sito ufficiale della Festa dell'Inquietudine*



[www.circoloinquieti.it](http://www.circoloinquieti.it)  
*Chi siamo, Storia, Eventi del Circolo degli Inquieti*



[www.slideshare.net/inquieti](http://www.slideshare.net/inquieti)  
*Presentazioni*

[www.slideshare.net/event/festa-inquietudine-2010](http://www.slideshare.net/event/festa-inquietudine-2010)  
*Evento Festa dell'Inquietudine 2010*



[lacivetta.wordpress.com](http://lacivetta.wordpress.com)  
*Blog del Circolo degli Inquieti*



<http://twitter.com/Inquietus>  
*Twitter microblogging*



[Circolo degli Inquieti](#)  
*Profilo Facebook del Circolo degli Inquieti*



<http://www.scribd.com/inquietus>  
*Scribd - Documenti*



[www.inquietudo.wetpaint.com](http://www.inquietudo.wetpaint.com)  
*Wiki: Attività pre/post Festa Inquietudine*

[www.inquietamente.wetpaint.com](http://www.inquietamente.wetpaint.com)  
*Wiki: Progetti Inquietamente*



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